RELATIONSHIP BETWEEN IN-SERVICE MATHEMATICS TEACHERS' MOTIVATIONAL AND EMOTIONAL ORIENTATIONS AND KNOWLEDGE IN STATISTICS

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Teachers' professional competence is understood to include both cognitive and affective aspects. In the field of statistics, however, studies that address the relationship between in-service teachers' orientations (affective aspect) and their knowledge (cognitive aspect) are scarce, and studies with prospective teachers yielded contradictory results in this regard. Accordingly, we surveyed 88 in-service mathematics teachers about their motivational and emotional orientations regarding teaching statistics, tested their basic statistical knowledge, and used linear mixed-effects models to analyze the relationship between orientations and knowledge. The results indicated that teachers with high self-efficacy showed higher statistical knowledge than less self-effective teachers, and that anxious women performed better than less anxious female teachers. This demonstrates the close relationship between the cognitive and affective aspects of in-service statistics teachers. Therefore, in order to develop professionally competent teachers, it seems worthwhile to address teachers' fears and to strengthen their self-efficacy already during their teacher training in statistics.

INTRODUCTION

Statistics teachers serve as multipliers of statistical literacy. To fulfil this role, they need *professional competence* to teach statistics effectively. In accordance with the *COACTIV* framework, we thereby understand teachers' professional competence to include affective aspects in addition to the cognitive aspect of professional knowledge (Baumert & Kunter, 2013). In general, the affective domain can be divided in terms of more cognitive, motivational, and emotional orientations (Hannula, 2012)—components that are considered significant in the performance of the profession and, relatedly, in its impact on learners (Schumacher, 2017; Groth & Meletiou-Mavrotheris, 2018). In recent studies of teachers' orientations and knowledge in the field of statistics education, there is a strong tendency to investigate prospective teachers rather than in-service teachers. However, research with prospective teachers is limited because these studies are not able to provide adequate information about the teachers' competence in relation to their teaching practice (Eichler & Zapata-Cardona, 2016). Such information, however, could be useful for the development of teacher training in statistics.

In particular, little is known about the relationship between the teachers' orientations regarding statistics and their statistical content knowledge (Eichler & Zapata-Cardona, 2016; Groth & Meletiou-Mavrotheris, 2018). Studies with prospective teachers yielded contradictory results in this regard: Hannigan et al. (2013) found no significant correlations whereas Nasser (2004) as well as Estrada and Batanero (2008) found weak and Zientek et al. (2011) even found moderate relationships between orientations and knowledge in statistics. All four studies used modifications of the *Survey of Attitudes Toward Statistics* (SATS; Schau et al., 1995) to capture cognitive, motivational, and emotional orientations, but they used different tests for measuring knowledge, which could provide an explanation for the different results. First results regarding in-service teachers were provided by a pilot study by Schumacher (2017), who found moderate correlations between teachers' orientations (i.e., self-efficacy, joy, anxiety, and anger) and knowledge regarding statistics. In addition, Nasser (2004) revealed with structural models that for prospective teachers, more positive orientations regarding statistics were related to statistical knowledge. Zientek et al. (2011) showed on the basis of a multiple regression model that different facets of prospective teachers' orientations regarding statistics (i.e., feelings, cognitive competence, value, difficulty, interest, and effort) explained their statistical knowledge.

The aim of our study was to complement existing research by analyzing the relationship between *in-service* mathematics teachers' orientations and their content knowledge in statistics. This focus on teaching practice is also reflected in our understanding of orientations in this study: we intentionally focus on *orientations regarding teaching statistics* instead of *orientations regarding*

In: R Helenius, E Falck (Eds.), *Statistics Education in the Era of Data Science*
Proceedings of the Satellite conference of the International Association for Statistical Education (IASE),
Aug-Sept 2021, Online conference. iase-web.org/Conference_Proceedings.php

statistics, even though the field has not yet made a strong distinction between these two approaches (Groth & Meletiou-Mavrotheris, 2018). In addition, we understand content knowledge at a level that does not go beyond school content. For our study, we selected facets of orientation from *BeSt Teacher*, an instrument that allows both orientations and knowledge to be assessed in the same framework. In particular, we investigated teachers' (1) *self-efficacy* (i.e., motivational orientation) to record teachers' belief in their ability to cope with lesson content (Bandura, 2010); (2) *joy* (i.e., emotional orientation) to record teachers' response to good teaching (Emmons, 2020); (3) *anxiety* (i.e., emotional orientation) to record teachers' worries and tension when teaching (Zeidner, 1991). Our study addresses the following research question: *Are in-service teachers' motivational and emotional orientations regarding teaching statistics related to their statistical content knowledge?* As previous studies with prospective teachers have yielded inconsistent results regarding the relationship between orientations and knowledge, we analyze this question exploratively: we conducted a model selection to identify facets of in-service teachers' orientation that are related to their statistical content knowledge.

METHOD

Participants

We investigated $N = 88$ Colombian in-service mathematics teachers (34 female, 53 male, 1 did not specify). They were 24–59 years old ($M = 38.1$, $SD = 8.5$) and had 1–37 years of teaching experience ($M = 10.5$, $SD = 6.9$). Female and male teachers did not differ significantly in their age or their teaching experience. The sample consisted of teachers teaching all grade levels in private and public schools, in urban as well as rural areas.

Instruments

For our investigations, we used the *BeSt Teacher* framework, which is a validated instrument to measure in-service teachers' professional competence in descriptive statistics (Schumacher, 2017). To assess motivational and emotional orientations regarding teaching statistics, the *BeSt Teacher* orientation scales were slightly modified to capture orientations regarding not only descriptive statistics, but statistics in general. The teachers' motivational orientation regarding teaching statistics was assessed by their *self-efficacy*, introduced by the stimulus "How confident do you feel in statistics? Please estimate in advance how confident you are in being able to solve tasks on the following topics". The scale consisted of seven items (e.g., "tasks concerning absolute and relative frequencies") that were assessed on 4-point Likert-scales (i.e., 1 = "unconfident", to 4 = "confident"), Cronbach's $\alpha = 0.91$. The teachers' emotional orientation was assessed by the *joy* and *anxiety* they feel teaching statistics, introduced by the stimulus "How do you feel about teaching statistics? Please indicate how much you agree with the following statements". Each scale consisted of four items (e.g., *joy*: "In general, I enjoy teaching statistics"; *anxiety*: "When teaching statistics, I am tense and nervous in general.") that were assessed on 4-point Likert-scales (1 = "I totally disagree", to 4 = "I totally agree"). For *joy*, $\alpha = 0.94$; for *anxiety*, $\alpha = 0.78$. Thus, for self-efficacy and joy, higher scores represent better orientations, while for anxiety, lower scores represent better orientations. To assess the teachers' *statistical content knowledge*, we used a selection of slightly modified items from the *BeSt Teacher* content knowledge test. In total, our test consisted of 16 items on various school-relevant concepts of descriptive statistics (i.e., absolute and relative frequencies, mean, median, boxplot) in different answer formats (i.e., single choice, multiple choice, numerical open-ended). The evaluation of the participants' answers was dichotomous (i.e., correct vs. incorrect), $\alpha = 0.64$.

Procedure

All teachers were participants in a four-hour teacher training on teaching statistics in Medellín, Colombia, taught by the first author of this paper. The training and thus the recruitment of study participants was organized by a division of the local Ministry of Education. Prior to the training, the teachers took part in the cross-sectional study on a voluntary basis and without reimbursement. They were informed about the purpose of the study and were asked for their informed consent. The questionnaire was presented in paper-based format in Spanish—the native language in Columbia. The questionnaire surveyed demographics data, motivational and emotional orientations regarding teaching statistics, and statistical content knowledge.

Data and Statistical Analysis

All analyses were conducted in *R* (R version 4.0.3; R Core Team, 2020). We studied the relationship between teachers' motivational and emotional orientations regarding teaching statistics and their statistical content knowledge with linear mixed models (LMMs). As we analyzed the research question on an exploratory basis, we aimed to find the model that best approximates teachers' *statistical content knowledge* (outcome variable), more specifically, the model that best estimates teachers' probability of getting the correct answer to a task of average difficulty in the statistical content knowledge test. Therefore, we conducted an automated model selection with the function *dredge* from the *MuMIn* package (R package version 1.43.17; Bartón, 2020), which returns the model with a subset of fixed effects of a given global model that has the minimum AIC-value. The global linear mixed model as the starting point of our analyses—and hence each candidate model in the model selection—included random intercepts for *teacher* and *item* to describe general differences between *teachers'* knowledge and between the difficulty, type, and content of *items*. In addition, the global LMM consisted of the fixed effects predictors *self-efficacy*, *joy*, and *anxiety* regarding teaching statistics (metric predictors standardized at the sample mean), *gender* (dichotomous factor with male as the baseline), *age*, *teaching experience* (metric predictors standardized at the sample mean) and the interaction effects of *gender* with *self-efficacy*, *joy*, and *anxiety*. As the evaluation of the outcome variable *statistical content knowledge* was dichotomous (i.e., correct vs. incorrect answer), we resorted to logistic regression. Accordingly, we report the odds ratio as an estimate for each predictor in the resulting model. We further report two relevant summarizing statistics of linear mixed-effects models: the *marginal* and the *conditional* R^2-value, which give the variance explained by the fixed effects only, and the entire model including random effects, respectively, thus providing values for the goodness-of-fit of the model (Nakagawa & Schielzeth, 2013). Furthermore, it is of interest to observe specific changes in the variance component *teacher*, which captures deviations attributable to differences between teachers in general. When adding (*teacher*-related) fixed effects to the intercept-only model, this variance may be partially explained by the predictors and thus reduce, which is quantified by the *Proportion Change in Variance* (PCV).

RESULTS

We investigated teachers' motivational and emotional orientations regarding teaching statistics as well as their statistical content knowledge. The intercorrelations of teachers' self-reported *self-efficacy* (motivational orientation, $M = 2.97$, $SD = 0.77$), *joy* (emotional orientation, $M = 3.18$, $SD = 0.63$), and *anxiety* (emotional orientation, $M = 2.11$, $SD = 0.94$) regarding teaching statistics and their performance in a *statistical content knowledge* test are given in Table 1.

Table 1. Intercorrelations of motivational and emotional orientations regarding teaching statistics and correlations with statistical content knowledge

Measure	1	2	3	4
1 Self-efficacy	-	-	-	-
2 Joy	0.53***	-	-	-
3 Anxiety	-0.33**	-0.19	-	-
4 Statistical content knowledge	0.40***	0.09	-0.07	-

Note. Levels of significance: ** $p < .01$, *** $p < .001$

Significant linear relationships exist between *self-efficacy* and all measures examined, whereas *joy* and *anxiety* do not correlate with each other or with *statistical content knowledge*. To answer the research question, we performed an automated model selection to reveal the relationship between teachers' *self-efficacy*, *joy*, and *anxiety* and their *statistical content knowledge*, while controlling for their *gender*, *age*, and *teaching experience*. As depicted in Table 2, the LMM resulting from the model selection process shows that both teachers' motivational and emotional orientations are related to their statistical content knowledge.

Table 2. Linear mixed model estimating in-service teachers' statistical content knowledge

Fixed Effects	*Odds Ratio*	*Standard Error*
Self-efficacy	1.44**	0.11
Anxiety	0.95	0.12
Age	0.78*	0.10
Gender (male → female)	0.66*	0.20
Gender (male → female) × Anxiety	1.60*	0.23
Random Effects	*Variance*	*PCV*
Teacher	0.38	38.3%
Item	1.41	-
Model Characteristics		
Observations (Teachers / Items / Total)	81 / 16 / 1296	
Marginal R^2	4.4%	
Conditional R^2	38.1%	

Note. The LMM results from the model selection process based on the global model with predictors *self-efficacy*, *joy*, and *anxiety* regarding teaching statistics, *gender*, *age*, *teaching experience*, and the interaction effects of *gender* with *self-efficacy*, *joy*, and *anxiety* as fixed effects, as well as *teacher* and *item* as random effects. Levels of significance: * $p < .05$, ** $p < .01$

Before answering the research question, we first describe the LMM resulting from the model selection process in detail. The estimates for the odds ratios of the predictors represent differences in *statistical content knowledge* between teachers with different degrees of *self-efficacy* (motivational orientation) and *anxiety* (emotional orientation) regarding teaching statistics—controlling for teachers' *age* and *gender*. A transformation of the intercept's odds ratio can be used to describe the estimated probability that a male teacher of average age, average self-efficacy, and average anxiety will correctly solve a task of average difficulty in the statistical content knowledge test (Table 2). This probability can be calculated as 74.5%, 95% CI [60.7, 84.6]. The PCV on the random intercept teacher states that 38.3% of the variance attributed to differences between the teachers' knowledge in general in the intercept-only model can be explained by the added predictors.

Table 2 reveals that teachers who reported higher *self-efficacy* showed significantly higher statistical content knowledge—controlling for the significant negative effect of increasing age and the significant gender effect in favor of male teachers. Thus, higher self-efficacy regarding statistics was positively related to teachers' content knowledge in statistics. Furthermore, Table 2 shows that female teachers who reported higher *anxiety* showed significantly higher statistical content knowledge than less anxious female participants—again controlling for the significant negative effect of increasing age. To ease interpretation of this interaction effect, it is visualized in Figure 1.

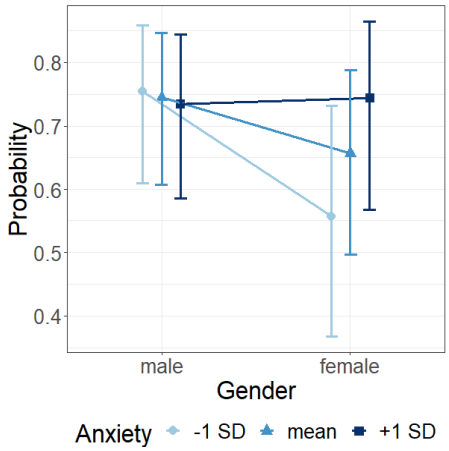

Figure 1. Interaction effect between gender and anxiety in the LMM for teachers with average self-efficacy and average age. Probabilities are estimated for teachers with reported anxiety 1 SD below, 1 SD above, and at the sample mean. Error bars represent 95% confidence intervals.

Thus, for women, higher anxiety about teaching statistics was positively related to their content knowledge in statistics. In summary, the linear mixed model sought on an exploratory basis to best approximate teachers' statistical content knowledge reveals that both teachers' motivational and emotional orientations are related to their statistical content knowledge.

DISCUSSION

The results of our study revealed that in-service teachers' motivational and emotional orientations are related to their statistical content knowledge. We used the model selection as an exploratory approach to finding the model that best estimates teachers' ability of providing the correct answer to a statistical task of average difficulty. The resulting model considered *self-efficacy* (motivational orientation) and *anxiety* (emotional orientation) regarding teaching statistics as explaining variables, while controlling for *age* and *gender*. *Joy* as the second facet for emotional orientation was not considered in the model as this scale received relatively high approval and only a small deviation from the sample and thus represented a rather homogeneous predictor. While the positive relationship between self-efficacy and knowledge has been observed similarly in previous studies (Schumacher, 2017), the result that female teachers with higher anxiety showed significantly higher statistical content knowledge was surprising. It contradicts previous findings from Schumacher's (2017) pilot study with in-service teachers based on the same scales, in which a high statistical knowledge was associated with a low level of anxiety. In our study, there was no such cross-gender correlation between anxiety and knowledge. It should also be mentioned that, in contrast to Schumacher's study, no general gender differences in anxiety occurred in our sample (M_f = 2.2, SD_f = 0.6, and M_m = 2.1, SD_m = 0.8, $t(81) = 0.43$, p = .67), even at the item level. Nasser (2004), who considered the relationship between knowledge and statistics anxiety of prospective teachers, has not performed gender analyses as 96% of her sample were female students. Considering the fact that we surveyed anxiety regarding *teaching* statistics, this high anxiety despite rather high statistical content knowledge for females might suggest that in particular female teachers perceive a gap between their content knowledge and their pedagogical content knowledge. Probably this facet is not sufficiently addressed in teacher training. It should be further investigated whether this is a stable effect. A notable secondary outcome is the significant difference in statistical knowledge with respect to gender. It was particularly surprising as male and female participants did not differ in their age or teaching experience or with regard to their self-reported self-efficacy, joy, and anxiety. Furthermore, a higher age was negatively related to statistical content knowledge. This can be explained by the fact that statistics only recently found its place in school curricula and in teacher training. Some teachers never even had statistical training (Ben-Zvi & Garfield, 2004; Schumacher, 2017). Teaching experience was not a significant predictor, yet highly correlated with age (r = .67, p < .001, 95% CI [.54, .78]). This suggests that teachers acquire statistical content knowledge essentially at university, which underpins the importance of teacher training as preparation for the career. There were similar conclusions in the *COACTIV* study, which revealed that mathematics teachers' content knowledge develops primarily in university and then stagnates or diminishes as they enter the teaching profession (Kleickmann et al., 2013). Looking ahead, the identified relationships between teachers' motivational and emotional orientations and their statistical content knowledge could be due to reciprocal causal relationships. Therefore, it might be worthwhile to address teachers' fears when teaching statistics and to strengthen their self-efficacy. However, our study covers only a selection of facets of orientations regarding teaching statistics. As a validated framework for the joint assessment of orientations and knowledge in statistics (currently only available in German), the *BeSt Teacher* framework (Schumacher, 2017) forms a solid basis to investigate additional facets of orientations and their relationship to knowledge in further studies. Such studies, particularly with in-service rather than pre-service teachers, are needed to obtain a more comprehensive picture of teachers' professional competence in statistics.

Altogether, the study provides new insights into in-service teachers' professional competence and thus contributes to basic research in statistics education. First, the results revealed existing relationships between orientations and knowledge in statistics. Accordingly, teacher training should not only address the statistical content but should focus also on developing positive orientations and pedagogical content knowledge. Second, the findings provide evidence that teacher training in university lays the groundwork for teachers' statistical content knowledge and should therefore be given higher priority in pre-service education of mathematics teachers.

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